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APPROXIMATE DETERMINATION OF JET CONTOURS NEAR THE EXIT OF AXIALLY SYMMETRICAL NOZZLES AS A BASIS FOR PLUME MODELING

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Army Missile Command
Redstone Arsenal, Alabama

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TECHNICAL REPORT
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18 August 1972 Technical Report No. RD-72-14

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DA Project No. 1M262303A214

AMC Management Structure Code No. 632303.11.21405

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ABSTRACT

A simple, approximate method is presented for rapid determination (utilizing mini-computers) of plume shapes produced by jet expansion from axially symmetric nozzles. The analysis is based on concepts developed by Johannesen and Meyer. It is shown how the method for plume shape determination can be utilized to model at least the geometric aspects of prototype plumes as well as to account for significant inviscid and viscid aspects of the base flow problem. A calculation procedure and numerical examples are presented to guide the reader in applying the modeling technique.

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SYMBOLS

a^*(ft/sec)	critical acoustic velocity
C_1(-)	constant of integration in Eq. (5), determined from Eq. (13)
M(-)	Mach number
M^*(-)	critical Mach number
R(-)	polar coordinate radius vector (dimensionless) (exit radius of nozzle = 1)
r_c(-)	initial radius of curvature of expanded free jet boundary
u(ft/sec), v(ft/sec)	velocity components
β_o(°)	constant of integration in Eq. (6), determined by Eq. (14)
γ(-)	specific heat ratio
ϕ(°)	polar angle
λ(-)	$[(\gamma - 1)/(\gamma + 1)]^{\frac{1}{2}}$
η(-)	$\lambda(\phi + \beta_o)$, auxiliary angle
θ(°)	streamline angle
μ(°)	Mach angle
ω(°)	Prandtl-Meyer streamline turning angle subscript

Subscripts

F	conditions at final expansion fan line as $R \to 0$
L	conditions at initial expansion fan line as $R \to 0$
J	free jet boundary as $R \to 0$
M	model
P	prototype

I. INTRODUCTION

Plume-slipstream interactions are of importance in propulsion, stability, and guidance problems related to missile flight. Depending on design performance and flight profile, low pressures in base regions may impair propulsive efficiency by imposing drag penalties (base drag) and also generate there an unfavorable thermal environment (base heating). With increasing jet-to-ambient pressure ratio, the plume size increases, adverse base drag effects gradually disappear, and larger-than-ambient base pressures produce base thrust. This can eventually lead to flow separation from the missile afterbody, a situation which may be considered beneficial in view of possible drag reduction but disturbing if buffetting and loss of control result from such interactions. The combined effects of plume induced separation and angle of attack of the missile can give rise to destabilizing moments [1].

Although comprehensive computer programs allow the analysis of jet-slipstream interactions for supersonic flight velocities, they fail to give coverage for the transonic flight regime which is most vulnerable to the adverse effects of plume induced separation. Consequently, the results of well planned experiments continue to serve as the main source for guidance.

The observation that transonic flows past afterbodies are not capable of large angular deflections in negotiating adverse pressure gradients places much emphasis on the study of plume boundaries as they establish geometrical constraints on the transonic flow approaching the base.

The analysis of plume shapes as they are affected by nozzle configuration, propellants, and pressure ratio attracts much interest. Utilization of well established computer programs based on the method of characteristics for axially symmetric supersonic flows is here supplemented by a simple MINI-COMPUTER oriented analysis based on earlier work by Johannesen and Meyer [2].

II. OBJECTIVES

The method presented here shall allow the rapid determination of plume shapes produced by jet expansion from axially symmetric nozzles. The effects of nozzle geometry (area ratio--or nominal exit Mach number and nozzle wall shape--divergence angle) propellant composition (γ, specific heat ratio) and overall pressure ratio (M_F or $(p/P_o)\big|_{M_F}$) are to be considered.

The approximate analytical method shall furnish a rational basis for plume-modeling.

III. ANALYSIS

The analysis follows the concepts developed by Johannesen and Meyer [2] in expressing the flow field near the centered expansion at the nozzle exit in the form of series expansions with respect to the radius vector R (Fig. 1). The velocity components thus are:

$$u = u_o(\phi) + u_1(\phi)\ R + O(R^2) \tag{1}$$

$$v = v_o(\phi) + v_1(\phi)\ R + O(R^2) \tag{2}$$

and after substitution into the conservation equations for axially symmetrical potential flow, one arrives at a system of equations for the yet unknown functions $u_o(\phi)$, $u_1(\phi)$, $v_o(\phi)$, and $v_1(\phi)$ which can, after comparing terms of equal order, be solved for given initial and final boundary conditions. Two types of solutions appear for the two types of flow regions A and B (Fig. 2):

1) For the centered expansion $\phi_L \leqslant \phi \leqslant \phi_F$ (Region A is identified in Fig. 2, a modified Prandtl-Meyer fan), one obtains

$$\frac{u_o(\phi)}{a^*} = \frac{1}{\lambda} \sin\left[\lambda(\phi + \beta_o)\right] \tag{3}$$

$$\frac{v_o(\phi)}{a^*} = \cos\left[\lambda(\phi + \beta_o)\right] \tag{4}$$

$$\begin{aligned} \frac{u_1(\phi)}{a^*} = -\frac{1}{2\lambda} (\cos\eta)^{\{[3\gamma-1]/[2(\gamma-1)]\}} (\sin\eta)^{\frac{1}{2}} \\ \cdot \Big\{[I_1(\lambda) - \lambda I_3(\eta)] \cos\beta_o \\ + [I_2(\eta) + \lambda I_4(\eta)] \sin\beta_o + C_1\Big\} \end{aligned} \tag{5}$$

where

$$\eta = \lambda(\phi + \beta_o) \tag{6}$$

and

$$I_1(\eta) = \int_{\eta_L}^{\eta} \cos\left(\frac{\eta}{\lambda}\right) (\sin\eta)^{-\frac{1}{2}} (\cos\eta)^{-(1/2\lambda^2)}\, d\eta$$

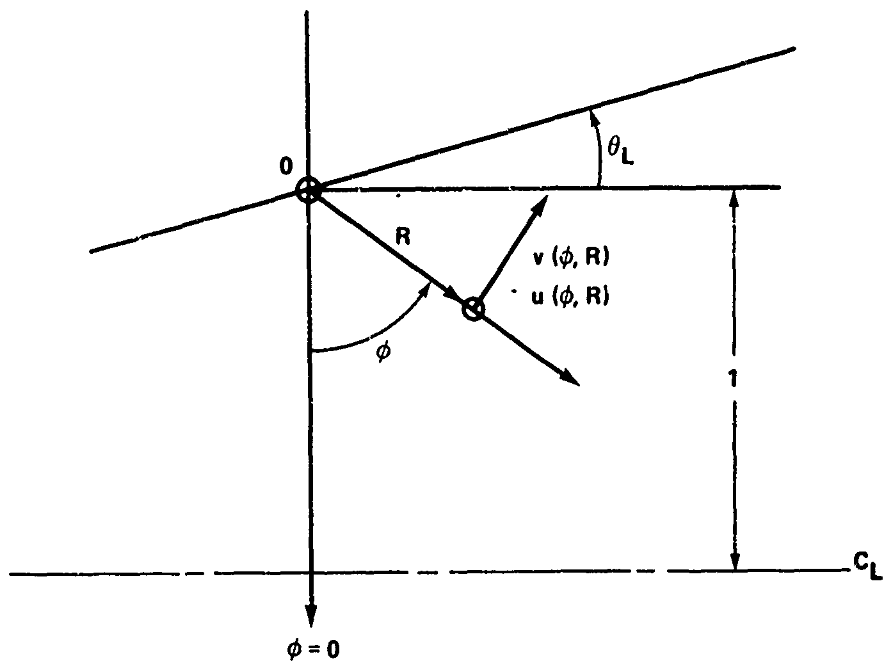

Figure 1. Coordinate System and Velocity Components

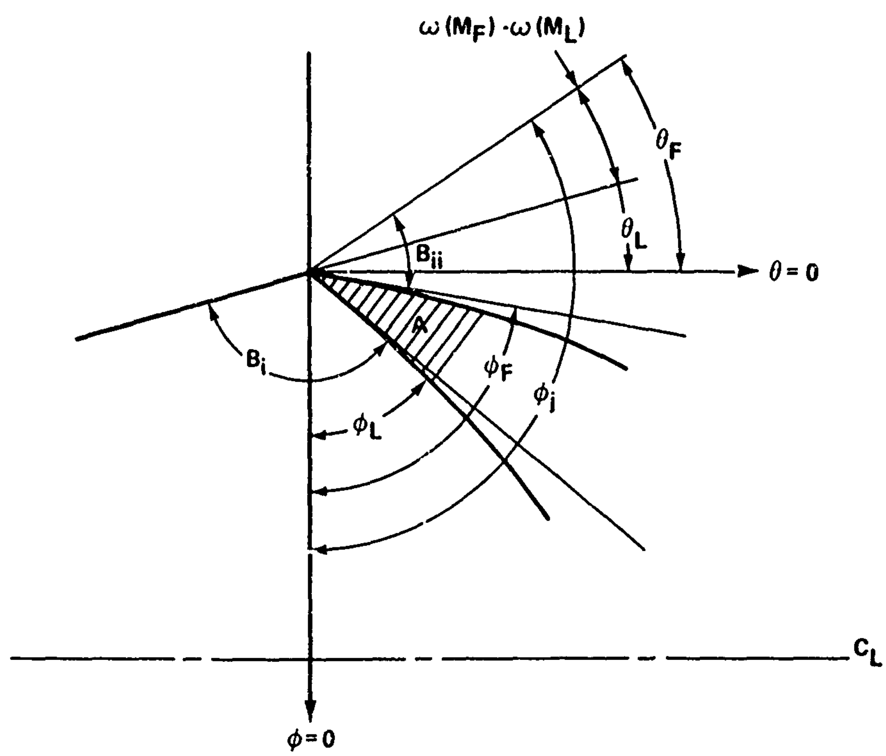

Figure 2. Flow Regions

$$I_2(\eta) = \int_{\eta_L}^{\eta} \sin\left(\frac{\eta}{\lambda}\right)(\sin\eta)^{-\frac{1}{2}}(\cos\eta)^{-(1/2\lambda^2)}\,d\eta$$

$$I_3(\eta) = \int_{\eta_L}^{\eta} \sin\left(\frac{\eta}{\lambda}\right)(\sin\eta)^{-3/2}(\cos\eta)^{\{[\gamma-3]/[2(\gamma-1)]\}}\,d\eta$$

$$I_4(\eta) = \int_{\eta_L}^{\eta} \cos\left(\frac{\eta}{\lambda}\right)(\sin\eta)^{-3/2}(\cos\eta)^{[\gamma-3]/[2(\gamma-1)]}\,d\eta$$

$$\frac{v_1(\phi)}{a^*} = \frac{1}{2}\frac{u_1'}{a^*} \tag{7}$$

while

$$\frac{u_1'}{a^*} = \frac{\frac{v_o}{a^*}\left[\frac{v_o}{a^*}\sin\phi - \frac{u_o}{a^*}\cos\phi\right] + \frac{v_o}{a^*}\frac{u_1}{a^*}\left[1 - \frac{3\gamma-1}{\gamma+1}\left(\frac{u_o}{v_o}\right)\right]^2}{2\frac{u_o}{a^*}} \tag{8}$$

2) Outside the centered wave region one arrives at relations which allow establishing boundary conditions from matching solutions at the lower (ϕ_L) and final (ϕ_F) Mach lines of the centered fan.

(a) The approaching flow yields (Region B is identified in Fig. 2 and shown in Fig. 3).

$$\frac{u_o(\phi_L)}{a^*} = M_L^* \cos\mu_L \tag{9}$$

$$\frac{v_o(\phi_L)}{a^*} = M_L^* \sin\mu_L \tag{10}$$

$$\frac{u_1(\phi_L)}{a^*} = M_L^*\left\{\sin 2\mu_L\left[\left.\frac{d\theta}{dR}\right|_{R=0} - \frac{1}{M_L^*}\left.\frac{dM^*}{dr}\right|_{R=0}\cot\mu_L\right] - \sin\theta_L\sin^2\mu_L\right\} \tag{11}$$

accounting for the flow conditions upstream of the nozzle lip, in particular, for the wall curvature $(d\theta/dR)\big|_{R\to 0}$ and the flow acceleration.

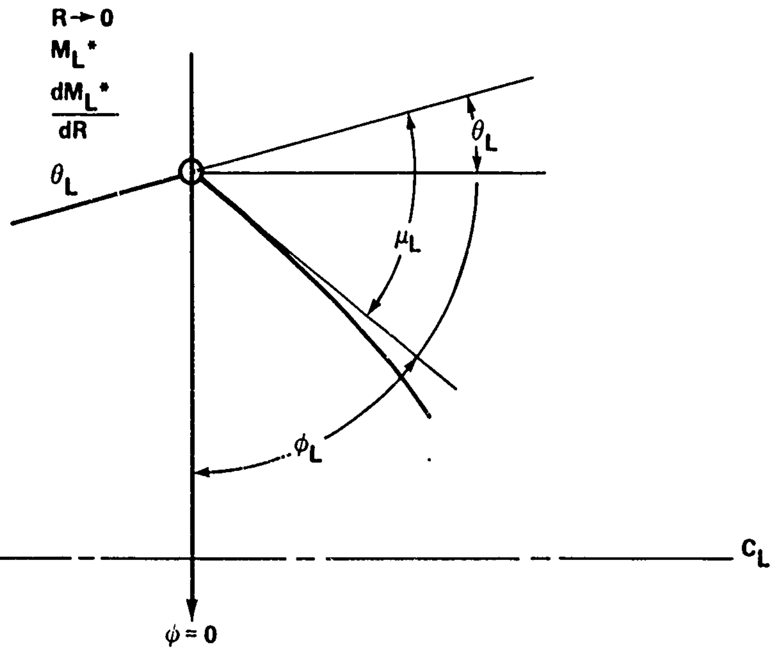

Figure 3. Boundary Conditions Imposed by Approaching Flow

For a conical nozzle, where $d\theta/dR = 0$, the assumption of source flow yields

$$\frac{u_1(\phi_L)}{a^*} = M_L^* \frac{4 \sin \theta_L \cos^2 \mu_L}{\frac{2}{\gamma - 1} \frac{\lambda^2 M_L^{*2}}{1 - \lambda^2 M_L^*} - 1} \tag{12}$$

so that C_1 in Eq. (5) can be evaluated

$$C_1 = - \frac{8 \lambda M_L^* \sin \theta_L \cos^2 \mu_L}{\left[\frac{2}{\gamma - 1} \frac{\lambda^2 M_L^{*2}}{1 - \lambda^2 M_L^{*2}} - 1\right] (\cos \eta_L)^{\{[3\gamma-1]/[2(\gamma-1)]\}} (\sin \eta_L)^{\frac{1}{2}}} \tag{13}$$

The constant β_o us determined through

$$\beta_o = \frac{1}{\lambda} \tan^{-1} \frac{\lambda}{\tan \mu_L} - \phi_L \tag{14}$$

Geometrical relations establish also

$$\phi_L = \theta_L - \mu_L + 90^{\circ} \tag{15}$$

in terms of the wall streamline angle θ_L and the Mach angle μ_L.

(b) The fully expanded flow satisfies (Region B is identified in Fig. 2 and shown in Fig. 4) for

$$\phi_F \leqslant \phi \leqslant \phi_j \quad (=\theta_F + 90^{\circ})$$

$$\left.\frac{d\theta}{dR}\right|_{R\to 0} - \frac{1}{M_F^*}\left.\frac{dM^*}{dR}\right|_{R\to 0} \cot \mu_F = \frac{1}{\sin 2\mu_F}\left[\sin \theta_F \sin^2 \mu_F + \frac{u_1(\phi_F)}{a^*}\frac{1}{M_F^*}\right] \tag{16}$$

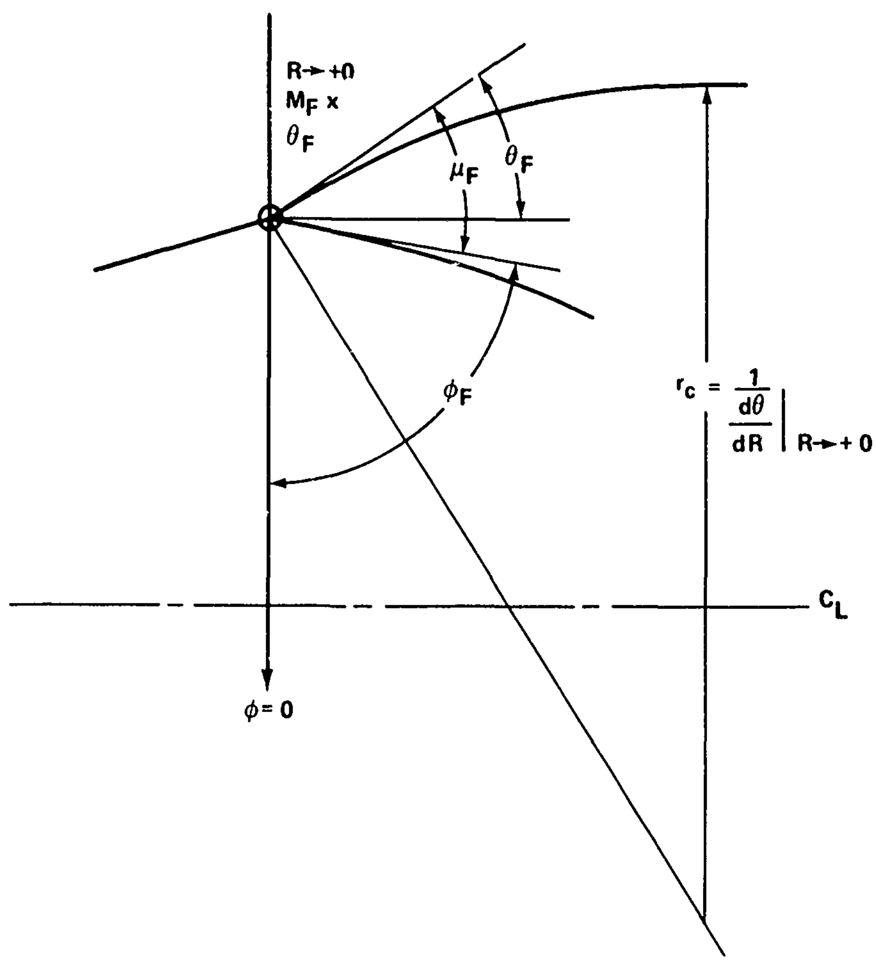

Figure 4. Boundary Conditions Imposed by Expanded Flow

where

$$\phi_F = \theta_F - \mu_F + 90^\circ; \qquad \theta_F = \theta_L + \omega(M_F) - \omega(M_L) \tag{17}$$

For the initial curvature of a free jet (plume) where dM*/dR = 0, one obtains, therefore,

$$\left.\frac{d\theta}{dR}\right|_{R\to 0} = -\frac{1}{\sin 2\mu_F}\left[\sin\theta_F \sin^2\mu_F + \frac{u_1(\phi_F)}{a^*}\frac{1}{M_F^*}\right] \tag{18}$$

The initial of curvature is

$$r_c = \frac{1}{\left.\frac{d\theta}{dR}\right|_{R\to 0}}$$

Obviously, Eq. (16) can also be used to determine the initial Mach number (or pressure) distribution along a wall boundary to which the nozzle flow expands. In this case $(d\theta/dR)|_{R\to 0}$ and θ_F are considered to be given and dM*/dR is to be solved for.

IV. PLUME MODELING

For any given input (γ, M_L, θ_L, and M_F), there results an approximate plume shape defined by the initial slope of the jet boundary θ_F and the (dimensionless) radius of curvature r_c.

1. MODELING LAWS

a. Geometric Modeling

If plume modeling is to be achieved, it will be necessary to match geometrically (within our degree of approximation)

$$\theta_{F_M} = \theta_{F_P} \tag{19}$$

and

$$r_{c_M} = r_{c_P} \quad . \tag{20}$$

One expects also that the following information is given for the prototype:

$$R_P, T_{o_P}, \gamma_P, M_L, \theta_{L_P}, \text{ and } M_{F_P} \quad .$$

For the model it shall be assumed that the propellant gas has been selected so that R_M and γ_M are determined. As we consider θ_{L_M}, M_{L_M}, and M_{F_M} as the parameters to be determined, the geometrical constraints (Eqs. (19) and (20)) must be supplemented by one additional condition: The proper modeling of the closure conditions for the wake is necessary.

b. Specifying Model Laws

Ideally, this specifying condition should properly account for the viscid aspects of the base flow problem in their interaction with the inviscid components. One must, on the other hand, realize that the comprehensive nature of the component model for jet-slipstream interactions in the vicinity of propulsive afterbodies [3] which apparently reflects physical reality, in essence requires two closure conditions to be satisfied. The first relates to the recompression ratio at the end of the wake (a dynamic condition), and the second condition calls for conservation of mass in the wake. With only one choice to be made for the closure of the model law, one must raise the question whether or not a logical choice can be made so that modeling can be accomplished in the most constructive manner. If this should prove impractical, one

could address oneself again to the comprehensive analytical model to augment the experimental modeling procedure, e.g., by a theoretically predetermined base bleed.

With these reservations, we shall now consider a number of simple specifying conditions for closure of the modeling law:

(1) If one adopts for this purpose the simple recompression model of the Chapman-Korst model (expecting that empirical corrections cancel out between model and prototype flows, it follows that

$$\left(1 - \phi_{d_M}^2 \lambda_M M^{*2}_{F_M}\right)^{(\gamma_M)/(\gamma_M-1)} = \left(1 - \phi_{d_p}^2 \lambda_p M^{*2}_{F_p}\right)^{(\gamma_p)/(\gamma_p-1)}$$

which, for $\phi_d^2 \lambda M^{*2} << 1$ and $\phi_{d_M} \simeq \phi_{d_p}$ reduces to

$$M^*_{F_M} = M^*_{F_p} \sqrt{\frac{\gamma_p}{\gamma_p + 1} \Big/ \frac{\gamma_M}{\gamma_M + 1}} \tag{21a}$$

(2) Matching the momentum ρu^2 of corresponding plume boundaries yields, since the pressure along plume surfaces are to be equal,

$$M_{F_M} = M_{F_p} \sqrt{\gamma_p/\gamma_M} \; .$$

(3) Matching the flux density ρu would result in the implicit relation

$$\frac{M^*_{F_M}}{1 - \lambda_M^2 M^{*2}_{F_M}} = \frac{M^*_{F_p}}{1 - \gamma_p^2 M^*_{F_p}} \sqrt{\frac{\left.\dfrac{\gamma}{(\gamma + 1) R T_o}\right|_p}{\left.\dfrac{\gamma}{(\gamma + 1) R T_o}\right|_M}}$$

which can be solved as a quadratic equation in $M^*_{F_M}$. It is interesting to note that there exists the possibility to explore the effect of stagnation temperature variation.

(4) Matching the supersonic inviscid streamline deflection-pressure rise relation on the basis of local linearization (weak shock approximation) requires

$$\frac{\gamma_p M^2_p}{\sqrt{M_p^2 - 1}} = \frac{\gamma_M M_M^2}{\sqrt{M_M^2 - 1}} \; . \tag{21d}$$

This, like other approximations, will impose certain mathematical and physical limits on the range of possible modeling.

2. CALCULATION PROCEDURE

We intend to model a prototype plume which results from

$$\gamma_p, M_{Lp}, \theta_{LP}, \text{ and } M_{Fp}$$

yielding an initial slope θ_F and an initial radius of curvature r_{c_P}. For a given model gas (R_M, T_{OM}, γ_M) selection of the proper specifying condition, e.g., Eq. (21a), will determine $M^*_{F_M}$.

Combining Eqs. (17) and (19) yields

$$\theta_{L_M} = \theta_{L_P} + \omega(M_{F_p}, \gamma_p) - \omega(M_{L_p}, \gamma_p) - \omega(M_{F_M}, \gamma_M) + \omega(M_{L_M}, \gamma_M) \tag{22}$$

where θ_{L_M} and M_{L_M} are unknown.

For any selected trial value of $M_{L_{M_i}}$ (there results now a $\theta_{L_{M_i}}$) so that the calculation procedure outlined in Section 3 will yield the curvature $r_{C_{M_i}}$. If a solution exists, it will be found by satisfying $r_{C_M} = r_{C_P}$ (Fig. 5). It is noteworthy that nozzle geometries, nozzle Mach numbers, and jet-to-ambient pressure ratios can differ widely between model and prototype, particularly also as a consequence of changing the specifying condition. On the other, even these wide parametric variations do not seem to impair the accuracy of matching the plume surface geometry. Thus, the present method affords a convenient approach to identifying and establishing the usefulness of the best suited specifying condition so that the entire plume-slipstream interaction is properly modeled.

3. NUMERICAL EXAMPLE

The prototype plume is defined by

$$\gamma_p = 1.2$$

$$\theta_{L_P} = 6^\circ$$

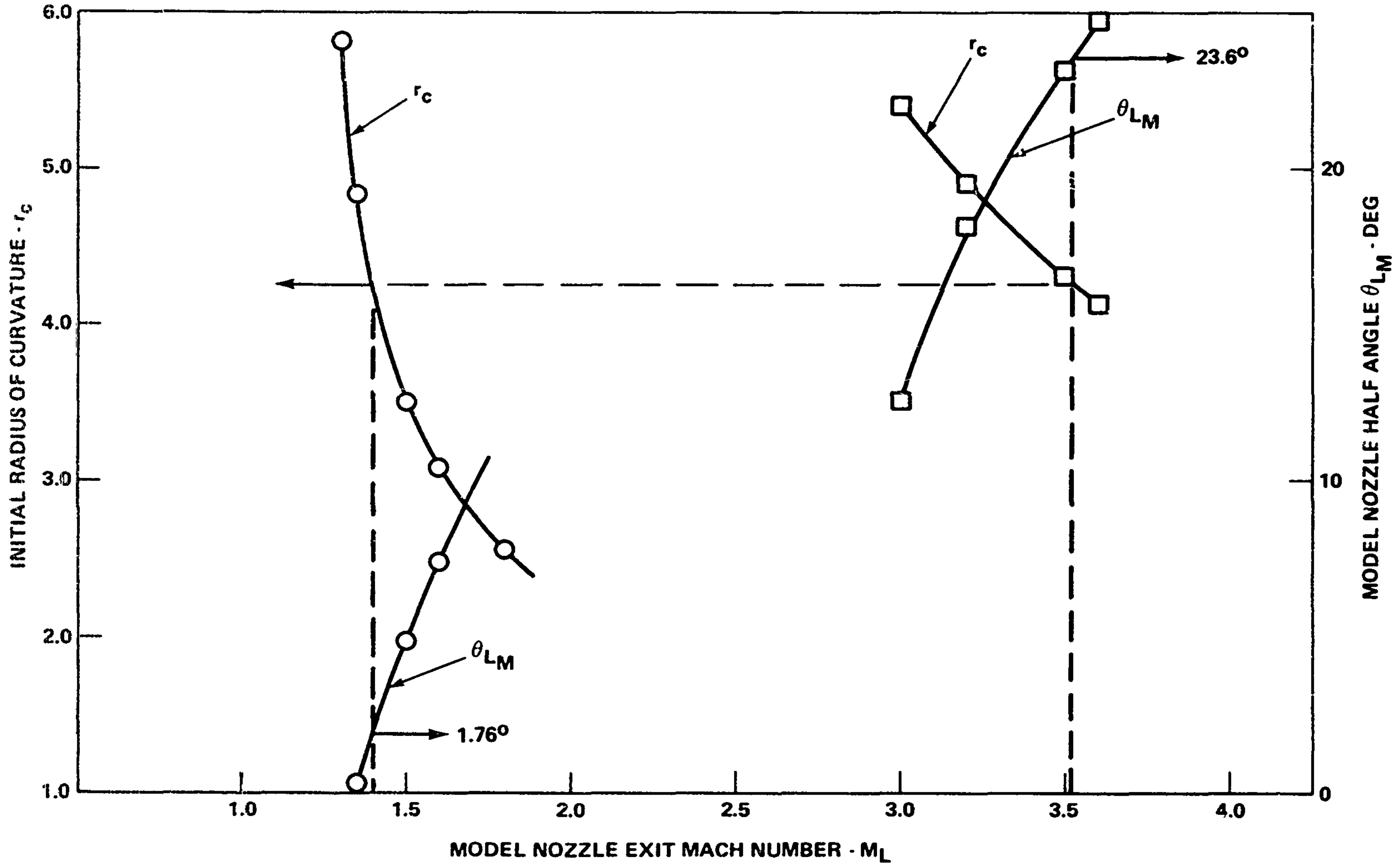

Figure 5. Trial-and-Error Solutions for Determining Model Parameters for Nozzle Exit Conditions

$$M_{L_p} = 2$$

$$M_{F_p} = 3 \qquad \left.\frac{p}{p_o}\right|_p = 0.02126 \ .$$

Modeling is to be done by air

$$\gamma_M = 1.4 \ .$$

Selection of different specifying conditions produces the following parameters:

TABLE I

	PROTOTYPE	MODEL, Eq. (21a)	MODEL, Eq. (21b)
γ	1.2	1.4	1.4
M_L	2.00	3.515	1.394
θ_L	6°	23.6°	1.75°
M_F	3.00	4.651	2.7775
θ_F	38.195°	38.2°	38.2°
r_c	4.2788	4.27	4.28

Shown in Figure 6 are the plume contours for the prototype flow and the two modeled flows as produced by the method of characteristics [3]. Agreement is excellent over the whole calculated range which assures the usefulness of the geometrical aspects of the modeling procedure independent of the selection of a specifying condition. Also plotted in Figure 6 is the circular arc approximation which forms the basis for the modeling procedure. The circular arc approximation, in its own rights, produces reasonable plume boundaries within the range of one nozzle exit radius.

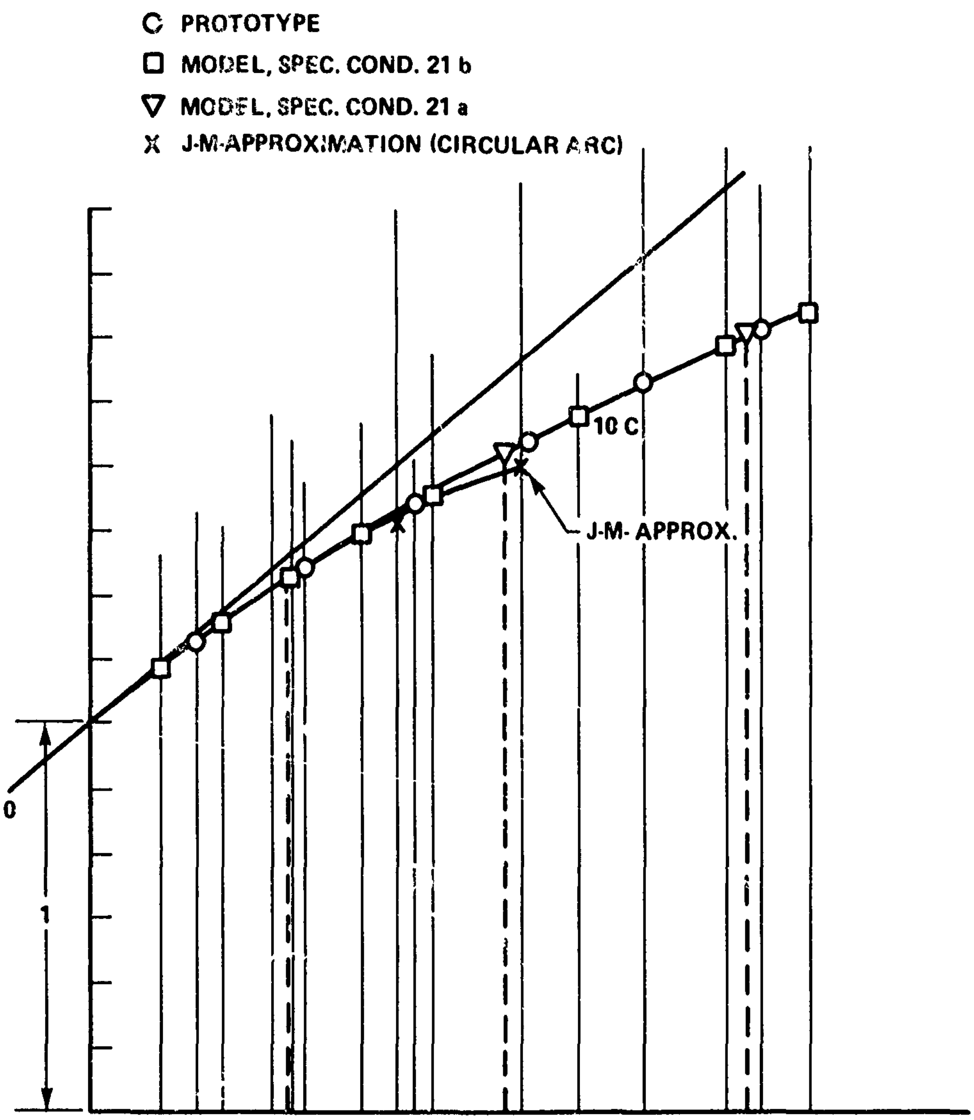

Figure 6. Plume Contours for Prototype and Model for Different Specifying Conditions

REFERENCES

1. Deep, R. A., J. H. Henderson, and C. E. Brazzel, "Thrust Effects on Missile Aerodynamics," US Army Missile Command, Redstone Arsenal, Alabama, Report No. RD-TR-71-9, May 1971.

2. Johannesen, N. H. and R. E. Meyer, "Axially-Symmetrical Supersonic Flow Near the Centre of an Expansion," The Aeronautical Quarterly, Vol. 2 (1950), pp. 127-142.

3. Addy, A. L., "Analysis of the Axisymmetric Base-Pressure and Base-Temperature Problem with Supersonic Interacting Freestream-Nozzle Flows Based on the Flow Model of Korst, et al., Part III: A Computer Program and Representative Results for Cylindrical, Boattailed, or Flared Afterbodies," US Army Missile Command, Redstone Arsenal, Alabama, Report No. RD-TR-69-14, February 1970.